A Check for Conceptual Understanding (ALG.CN.15)

Determine whether each statement is true or false. If it is false, identify why.

1. The sum of two complex numbers is always complex.

   True. Even when adding two complex conjugates, $(a+bi)+(a-bi)=2a$, the sum can still be written as a complex number, $2a+0i$.

2. The product of two complex numbers is sometimes complex.

   False. The product of two complex numbers is <u>always</u> complex. Even when multiplying two complex conjugates, $(a+bi)(a-bi)=a^2+b^2$, their product can still be written as a complex number $(a^2+b^2)+0i$.

3. The sum of two imaginary numbers is always imaginary.

   False. For example, $-2i+2i=0$. 0 is not imaginary.

4. There is no complex number that is equal to its complex conjugate.

   False. Every real number is equal to its complex conjugate.

5. $(\sqrt{-4})(\sqrt{-8})=4\sqrt{2}$

   False. $(\sqrt{-4})(\sqrt{-8})=-4\sqrt{2}$

6. All real numbers are imaginary numbers.

   False. The set of real numbers is not a subset of the set of imaginary numbers.

7. All real numbers are complex numbers.

   True. The set of real numbers is a subset of the set of complex numbers.

8. All imaginary numbers are complex numbers.

   True. The set of imaginary numbers is a subset of the set of complex numbers.

9. A rational number is a complex number.

   True. The set of rational numbers is a subset of the set of complex numbers.

10. Every complex number is a real number.

    False. Every real number is a complex number. (See #7)

11. π is a complex number.

    True. π can be written in $a+bi$ form as $\pi+0i$.

12. The real part of $\mathbf{12i}$ is $\mathbf{0}$.

    True. $12i$ can be written in $a+bi$ form as $0+12i$.

13. The square root of a negative number is an imaginary number.

True. By definition, $\sqrt{-b} = bi$.

14. The product of a complex number and its complex conjugate is always a real number.

True. $(a + bi)(a - bi) = a^2 + b^2$

15. $i^{40} + i^{41} + i^{42} + i^{43} + i^{44} = 1$

True. $1 + i - 1 - i + 1 = 1$

16. If $(a + bi)^3 = 8$, then $a^2 + b^2 = 4$.

True. There are three complex solutions for the equation $(a + bi)^3 = 8$. All three satisfy the equation $a^2 + b^2 = 4$.