Tangent Line Equations (CALC.DIF.05)

1. Write an equation of the tangent line to the graph of $f(x)$ at the given value of x.

$$f(x) = \frac{1}{2}x^4 - 3x + 6 \qquad x = 1 \qquad \boldsymbol{y - \frac{7}{2} = -(x - 1)}$$

2. Write an equation of the tangent line to the graph of $g(x)$ at the given value of x.

$$g(x) = \frac{1}{x} - \frac{1}{x^2} \qquad x = -2 \qquad \boldsymbol{y + \frac{3}{4} = -1/2\ (x + 2)}$$

3. Write an equation of the tangent line to the graph of $f(x)$ at the given value of x.

$$f(x) = x^2 \cdot \sin x \qquad x = \frac{\pi}{2} \qquad \boldsymbol{y - \frac{\pi^2}{4} = \pi\left(x - \frac{\pi}{2}\right)}$$

4. Write an equation of the tangent line to the graph of $g(x)$ at the given value of x.

$$g(x) = \frac{1}{x} - \sqrt{\cos x} \qquad x = \frac{\pi}{3} \qquad \boldsymbol{y - \left(\frac{3}{\pi} - \frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{6}}{4} - \frac{9}{\pi^2}\right)\left(x - \frac{\pi}{3}\right)}$$

5. Write an equation of the tangent line to the graph of $f(x)$ at the given value of x.

$$f(x) = \sqrt{x^2 + x} \qquad x = 1 \qquad \boldsymbol{y - \sqrt{2} = \frac{3\sqrt{2}}{4}(x - 1)}$$

6. Write an equation of the tangent line to the graph of $g(x)$ at the given value of x.

$$g(x) = x \cdot \ln x^2 \qquad x = 1 \qquad \boldsymbol{y = 2x - 2}$$

7. Write an equation of the tangent line to the graph of $g(x)$ at the given value of x.

$$g(x) = \sqrt{x} - \frac{1}{4}e^x \qquad x = \ln 16 \qquad \boldsymbol{y - 2\sqrt{\ln 2} + 4 = \left(\frac{1}{4\sqrt{\ln 2}} - 4\right)(x - \ln 16)}$$

8. Write an equation of the tangent line to the graph of $h(x)$ at the given value of x.

$$h(x) = (\ln x)^3 \qquad x = e^3 \qquad \boldsymbol{y - 27 = \frac{27}{e^3}(x - e^3)}$$

9. Write an equation of the tangent line to the graph of $f(x)$ at the given value of x.

$$f(x) = 2x + e^{2x} \qquad x = 0 \qquad \boldsymbol{y = 4x + 1}$$

10. Write an equation of the tangent line to the graph of $g(x)$ at the given value of x.

$$g(x) = x(e^{2x} - e^x) \qquad x = -1 \qquad \boldsymbol{y - \frac{1}{e} + \frac{1}{e^2} = -\frac{1}{e^2}(x + 1)}$$

11. Write an equation of the tangent line to the graph of $f(x)$ at the given value of x.

$$f(x) = x^4 - 4x^3 + 5x + 3 \qquad x = 1 \qquad \boldsymbol{y = -3x + 8}$$

12. Write an equation of the tangent line to the graph of $g(x)$ at the given value of x.

$$g(x) = \frac{1 + \sec x}{1 - \sec x} \qquad x = \frac{3\pi}{4} \qquad \boldsymbol{y - (2\sqrt{2} - 3) = \frac{2\sqrt{2}}{(1 + \sqrt{2})^2}\left(x - \frac{3\pi}{4}\right)}$$

13. Determine the point of tangency where the function has a horizontal tangent line.

$$f(x) = \ln\sqrt{\frac{e^{x-1}}{x+1}} \qquad \boldsymbol{\left(0, -\frac{1}{2}\right)}$$

14. Find k such that the line is tangent to the graph of the function.

$$f(x) = kx^2 \qquad y = -4x + 5 \qquad \boldsymbol{k = -\frac{4}{5}}$$

15. Find k such that the line is tangent to the graph of the function.

$$f(x) = kx^{2/3} \qquad y = -2x - 8 \qquad \boldsymbol{k = -6}$$

16. Find equations of the tangent lines to the graph of $p(x)$ that are parallel to the given line.

$$p(x) = 2x^3 - 5x^2 + 3x - 9 \qquad 21x - 3y = -25 \qquad \boldsymbol{y + 7 = 7(x - 2);\ y + \frac{287}{27} = 7\left(x + \frac{1}{3}\right)}$$

17. Find equations of the tangent lines to the graph of $f(x)$ that are parallel to the given line.

$$f(x) = \frac{x - 2}{x + 2} \qquad 8x - 2y = -13 \qquad \boldsymbol{y = 4x + 1; y = 4x + 17}$$

18. The given curve is called a **Witch of Agnesi**. Find an equation of the tangent line to this curve at the given point.

$$y = \frac{1}{1 + x^2} \qquad P\left(-2, \frac{1}{5}\right) \qquad \boldsymbol{y = \frac{4}{25}x + \frac{13}{25}}$$

19. Graph $f(x)$ and $g(x)$ in the same coordinate plane. Find equations of the two lines that are simultaneously tangent to both parabolas.

$$f(x) = -x^2 \qquad g(x) = x^2 - 2x + 5 \qquad \boldsymbol{y = 2x + 1; y = -4x + 4}$$

20. Show that the graph of the function does not have a horizontal tangent line.

$$f(x) = 5x + \cos x - 4 \qquad \boldsymbol{f'(x) = 5 - \sin x;\ f'(x) = 0 \text{ has no solutions}}$$