

## Implicit Differentiation (CALC.DIF.07)

Find  $dy/dx$  by implicit differentiation.

1.  $x^2y^2 - 4x^2 - 9y^2 = 0$

$$y' = \frac{4x - xy^2}{x^2y - 9y}$$

2.  $\ln xy - x^2 + y = 4$

$$y' = \frac{2x^2y - y}{x + xy}$$

3.  $-\cos(x - y) = x + y$

$$y' = \frac{\sin(x - y) - 1}{\sin(x - y) + 1}$$

4.  $\sqrt{x + y} = \frac{1}{x} + y$

$$y' = \frac{x^2 + 2\sqrt{x + y}}{2x^2\sqrt{x + y} - x^2}$$

5.  $\ln\sqrt{xy} - (x - y)^2 = 4$

$$y' = \frac{4x^2y - 4xy^2 - y}{4x^2y - 4xy^2 + x}$$

6.  $(x^2 - y)^2 = 2xy$

$$y' = \frac{2x^3 - 2xy - y}{x^2 + x - y}$$

7.  $4x^3 + \ln y^2 + y = 4x$

$$y' = \frac{4y - 12x^2y}{y + 2}$$

8.  $\frac{x}{y} + \frac{y}{x} = 4y$

$$y' = \frac{y^3 - x^2y}{xy^2 - x^3 - 4x^2y^2}$$

9.  $e^y \sin x = x - xy$

$$y' = \frac{e^y \cos x + y - 1}{e^y \sin x + x}$$

10.  $x \cos y + y \cos x = 1$

$$y' = \frac{y \sin x - \cos y}{\cos x - x \sin y}$$

Use implicit differentiation to write an equation of the tangent line to the graph at the given point.

11.  $x^2 + y^2 = 25$   $(-3, 4)$

$$y - 4 = \frac{3}{4}(x + 3)$$

12.  $xy = 10$   $(-5, -2)$

$$y + 2 = -\frac{2}{5}(x + 5)$$

13.  $x^2y - x^2 + 5x = -6$   $(-3, 2)$

$$y - 2 = \frac{1}{9}(x + 3)$$

14.  $x^2 + xy + y^2 = 9$   $(-3, 0)$

$$y = -2x - 6$$

15.  $\sin^2 x + \cos y = 1$   $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$$y - \frac{\pi}{3} = \frac{2\sqrt{3}}{3}\left(x - \frac{\pi}{4}\right)$$

16.  $e^{xy} - 2x = 0$   $(1, \ln 2)$

$$y - \ln 2 = (1 - \ln 2)(x - 1)$$

$$17. \sqrt{x} + \sqrt{y} = \frac{3}{4}xy \quad (4,1)$$

$$y - 1 = -\frac{1}{5}(x - 4)$$

$$18. y^2 = \ln xy \quad (e,1)$$

$$y = \frac{1}{e}x$$

$$19. x + y - 1 = \ln(x^2 + y^2) \quad (1,0)$$

$$y = x - 1$$

$$20. x^2y - xy + xy^2 = 4 \quad (2,1)$$

$$y - 1 = -\frac{2}{3}(x - 2)$$

Find  $d^2y/dx^2$  implicitly in terms of  $x$  and  $y$ .

$$21. 3x^4 - y^2 = 12$$

$$y'' = \frac{18x^2y^2 - 36x^6}{y^3}$$

$$22. x^2 + y^2 = 81$$

$$y'' = \frac{-x^2 - y^2}{y^3} = \frac{-81}{y^3}$$

$$23. y^3 = x^2$$

$$y'' = \frac{-2x^2}{9y^5}$$

$$24. x^2 - e^y - 4x = -20$$

$$y'' = \frac{2e^y + 16x - 4x^2 - 16}{e^{2y}} = \frac{64}{e^{2y}} - 1$$

$$25. xy - x^2 = -6$$

$$y'' = \frac{2y - 2x}{x^2} = \frac{-12}{x^3}$$

$$26. y^2 - x^2 + 5x = -10$$

$$y'' = \frac{4y^2 - 4x^2 + 20x - 25}{4y^3} = -\frac{65}{4y^3}$$

$$27. x^2y + 4x = 8$$

$$y'' = \frac{2x^2y + 32}{x^4}$$

$$28. xy - 3 = 3x + y^2$$

$$y'' = \frac{24}{(x - 2y)^3}$$