Separable Differential Equations (CALC.DEQ.04)

Find the general solution for each separable differential equation.

1. $\frac{dy}{dx} = \frac{12x^3}{4y - \sin y}$ $\quad$ $\mathbf{2y^2 + \cos y = 3x^4 + C}$

2. $y' = \frac{1}{12}x^2 y$ $\quad$ $\mathbf{y = Ce^{x^3/36}}$

3. $\frac{dy}{dx} = 3x\sqrt{y}$ $\quad$ $\mathbf{y = \frac{1}{16}(3x^2 + 2C)^2}$

4. $(e^y + 1)y' = 2 - \sec^2 x$ $\quad$ $\mathbf{e^y + y = 2x - \tan x + C}$

5. $y' = xe^y$ $\quad$ $\mathbf{y = -\ln\left(C - \frac{x^2}{2}\right)}$

6. $x + 2y\sqrt{x^2 - 4} \cdot y' = 0$ $\quad$ $\mathbf{y = C - \sqrt{x^2 - 4}}$

7. $xy' = 3(y - 2)$ $\quad$ $\mathbf{y = Cx^3 + 2}$

8. $\frac{dy}{dx} = xe^{x^2 - \ln y^2}$ $\quad$ $\mathbf{y^3 = \frac{3}{2}e^{x^2} + 3C}$

9. $\frac{dy}{dx} = e^{x - 2y}$ $\quad$ $\mathbf{y = \frac{1}{2}\ln(2e^x + 2C)}$

10. $\frac{2\ln x}{x} = y \cdot y'\sqrt{y^2 + 9}$ $\quad$ $\mathbf{\frac{1}{3}(y^2 + 9)^{3/2} = (\ln x)^2 + C}$

Find the particular solution that satisfies the initial condition.

11. $y \cdot y' - 5e^x = 10$ $\quad$ $y(0) = 2$ $\quad$ $\mathbf{y^2 = 10e^x + 20x - 6}$

12. $2y \cdot y' = 4\sin x$ $\quad$ $y\left(\frac{\pi}{4}\right) = \sqrt{2}$ $\quad$ $\mathbf{y^2 = -4\cos x + 2 + 2\sqrt{2}}$

13. $\frac{dy}{dx} = ye^{-x}$ $\quad$ $y(0) = e$ $\quad$ $\mathbf{y = e^{2 - e^{-x}}}$

14. $\sqrt{x} - \sqrt{y} \cdot y' = 0$ $\quad$ $y(9) = 1$ $\quad$ $\mathbf{y^{3/2} = x^{3/2} - 26}$

15. $y(2x - 1) + y' = 0$ $\quad$ $y(-3) = e$ $\quad$ $\mathbf{y = e^{x - x^2 + 13}}$

16. $y' = -2\tan y$ $\quad$ $y(\ln 2) = \frac{\pi}{2}$ $\quad$ $\mathbf{y = \sin^{-1}(4e^{-2x})}$

17. $y \cdot \ln x - xy' = 0$ $\quad$ $y(e^2) = 1$ $\quad$ $\mathbf{y = e^{(\ln x)^2/2 - 2}}$

18. $y\sqrt{4 - x^2} \cdot y' = x\sqrt{4 - y^2}$ $\quad$ $y(0) = 1$ $\quad$ $\mathbf{y^2 = 4 - \left(\sqrt{4 - x^2} + \sqrt{3} - 2\right)^2}$

19. $y' = xy\sin x^2$ $\quad$ $y(0) = \sqrt{e}$ $\quad$ $\mathbf{y = e^{-\cos(x^2)/2 + 1}}$

20. $y' = e^{y - x}(x - 1)$ $\quad$ $y(0) = 1$ $\quad$ $\mathbf{y = -\ln\left(x \cdot e^{-x} + \frac{1}{e}\right)}$