First-Order Linear Differential Equations (CALC.DEQ.05)

Find the general solution of each differential equation.

1. $y' + y = 3$ $\quad$ $y = 3 + Ce^{-x}$

2. $y' + \left(\frac{1}{x}\right)y = 9x + 4$ $\quad$ $y = 3x^2 + 2x + \frac{C}{x}$

3. $y' + y = e^{-x}$ $\quad$ $y = xe^{-x} + Ce^{-x}$

4. $y' + 4y = e^{4x}$ $\quad$ $y = \frac{1}{8}e^{4x} + Ce^{-4x}$

5. $y' - 2xy = e^{x^2}$ $\quad$ $y = xe^{x^2} + Ce^{x^2}$

6. $2xy' + y = 10\sqrt{x}$ $\quad$ $y = 5\sqrt{x} + \frac{C}{\sqrt{x}}$

7. $y' + y\cot x = 2\cos x$ $\quad$ $y = \cos x - C\cos x \cdot \csc^2 x$

8. $2xy' - 3y = 15x^3$ $\quad$ $y = 5x^3 + Cx^{3/2}$

9. $xy' = 4y + x^5\cos x$ $\quad$ $y = x^4\sin x + Cx^4$

10. $xy' + (2x - 3)y = 4x^4$ $\quad$ $y = 2x^3 + Cx^3e^{-2x}$

Find the particular solution that satisfies the initial condition.

11. $(1 + x)y' + y = \cos x$ $\quad$ $y(0) = 2$ $\quad$ $y = \frac{2 + \sin x}{x+1}$

12. $xy' - 4y = 6x^3$ $\quad$ $y(2) = 24$ $\quad$ $y = \frac{9}{2}x^4 - 6x^3$

13. $y' = 1 + x + y + xy$ $\quad$ $y(0) = 0$ $\quad$ $y = e^{x^2/2+x} - 1$

14. $xy' + 3y = \frac{\ln x}{x^2}$ $\quad$ $y(1) = 3$ $\quad$ $y = \frac{x\ln x - x + 4}{x^3}$

15. $(x^2 + 9)y' + xy = 3x$ $\quad$ $y(0) = 1$ $\quad$ $y = 3 - \frac{6}{\sqrt{x^2 + 9}}$

16. $\sin x \cdot y' + y\cos x = \sin(2x)$ $\quad$ $y\left(\frac{\pi}{2}\right) = 1$ $\quad$ $y = \sin x$

17. $y'\cos^2 x + y = 4$ $\quad$ $y(0) = 8$ $\quad$ $y = 4e^{-\tan x} + 4$

18. $2xy' - y = x^3 - x$ $\quad$ $y(25) = 100$ $\quad$ $y = \frac{1}{5}x^3 - x - 600\sqrt{x}$