Quadratic Functions in Standard Form (ALG.QUAD.01)

1. Use the quadratic function $f(x) = 5x^2 - 7x - 2$ to answer each part.

a. Determine the leading coefficient of the function.

b. What is the linear term?

c. What does the constant indicate?

d. Does the graph of f open upward or downward? Explain your answer.

e. How many x-intercepts will the graph of f have? Explain your answer.

2. Use the quadratic function $g(x) = -\frac{1}{3}x^2 + x - 6$ to answer each part.

a. Determine the leading coefficient of the function.

b. What does the leading coefficient of the function indicate for the graph of g?

c. What is the quadratic term?

d. Determine the y-intercept of the graph.

e. How many x-intercepts will the graph of g have? Explain your answer.

For each quadratic function, determine (i) the vertex, (ii) whether the vertex is a maximum or minimum value of the function, (iii) whether the parabola opens upward or downward, (iv) the domain and range, (v) the axis of symmetry, and (vi) on what intervals the graph of the function is increasing and decreasing.

3. $f(x) = x^2 - 4x$

4. $g(x) = -3x^2 + 5$

5. $f(x) = 9 - x^2$

6. $g(x) = 21 - 20x + 10x^2$

7. $f(x) = \frac{1}{2}x^2 - 5x + 9$

8. $g(x) = -0.75x^2 - 1.8x + 4.5$

9. $f(x) = -\frac{2}{3}x^2 + \frac{6}{5}x - \frac{8}{15}$

10. $g(x) = 0.1x^2 + 1.2x - 0.6$

Evaluate each quadratic function for the given values of x.

11.	$f(x) = x^2 - 5x + 2$	$x = 0$	$x = -2$	$x = -\frac{1}{2}$	$x = \sqrt{5}$
12.	$g(x) = 16 - x^2$	$x = 0$	$x = -4$	$x = 2.5$	$x = -2\sqrt{3}$
13.	$f(x) = 3x^2 - 7x + 2$	$x = 1$	$x = \frac{1}{3}$	$x = 0.75$	$x = \sqrt{6}$
14.	$g(x) = -\frac{3}{4}x^2 - \frac{3}{2}x + \frac{1}{3}$	$x = -2$	$x = \frac{8}{3}$	$x = -0.5$	$x = \frac{\sqrt{13}}{3} - 1$

In Exercises 15–20, match the quadratic function with its graph. The graphs are labeled (a), (b), (c), (d), (e), and (f).

15. $f(x) = x^2 - 4x + 3$

16. $f(x) = x^2 - 4x + 5$

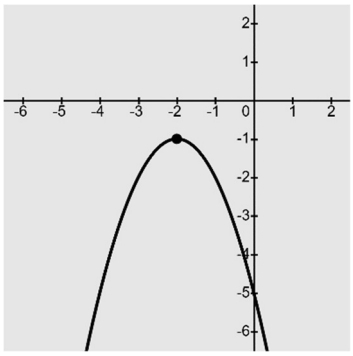
17. $f(x) = x^2 + 4x + 5$

18. $f(x) = -x^2 - 4x - 3$

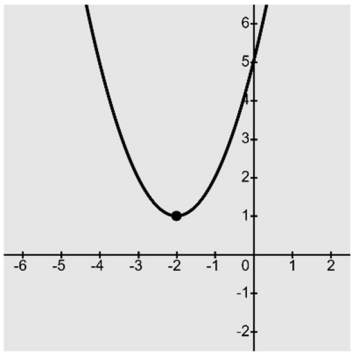
19. $f(x) = -x^2 + 4x - 5$

20. $f(x) = -x^2 - 4x - 5$

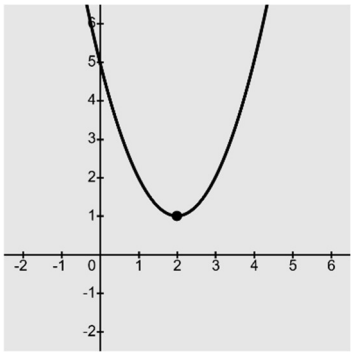
(a)



(b)



(c)



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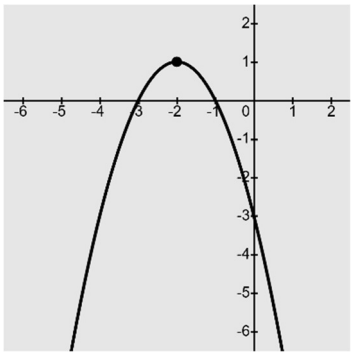

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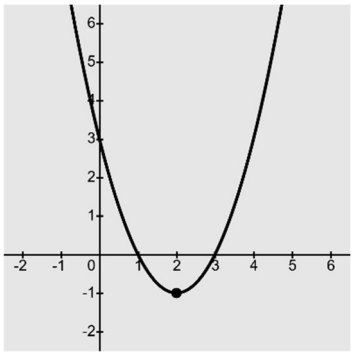
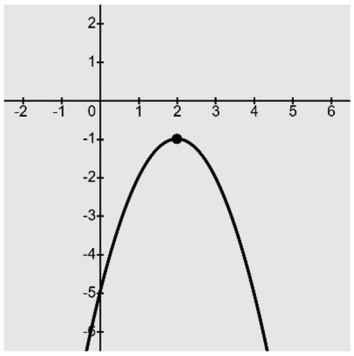


(f)



Convert each quadratic function to standard form.

21. $f(x) = (x - 2)^2 + 3$

22. $g(x) = (x + 6)^2 - 24$

23. $f(x) = (6x + 1)^2 - 1$

24. $g(x) = (2x - 5)^2 - 8$

25. $f(x) = -7\left(\frac{1}{2}x - 3\right)^2$

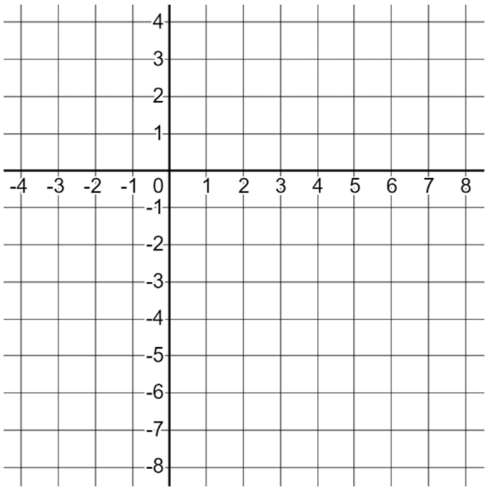
26. $g(x) = 4(x + 2)^2$

27. $f(x) = \frac{2}{3}(3x - 1)^2 + 6$

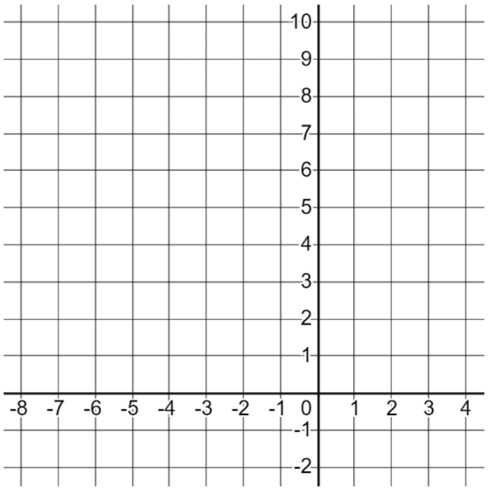
28. $g(x) = -\frac{1}{5}(x + 5)^2 + 7$

Graph each quadratic function by first finding its vertex and completing a table of values.

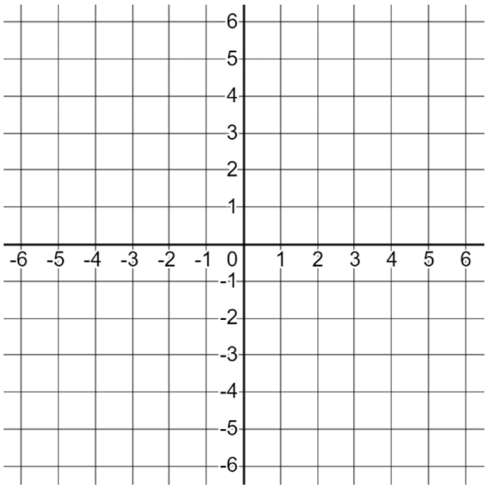
29. $f(x) = x^2 - 6x + 2$



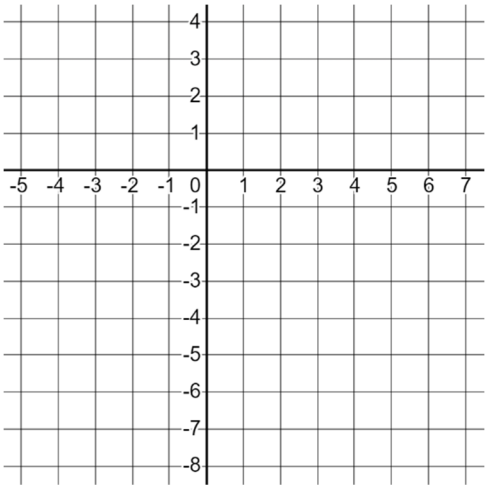
30. $g(x) = -x^2 - 4x + 5$



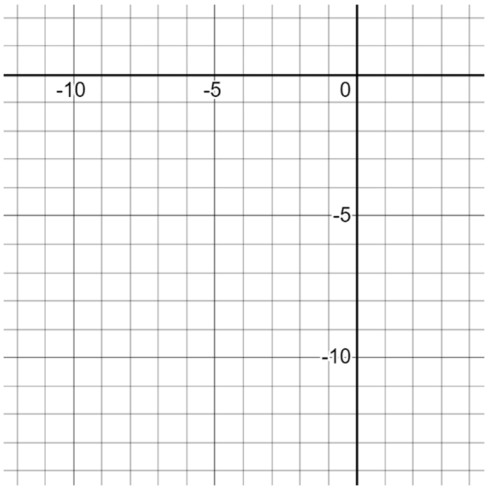
31. $f(x) = 3x^2 - 2x - 4$



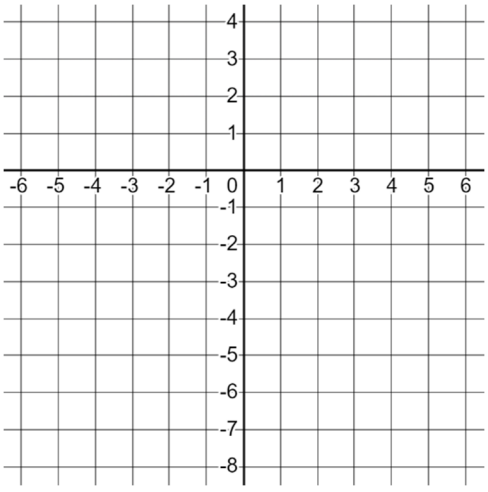
32. $g(x) = -5x^2 + 10x - 2$



33. $f(x) = 0.6x^2 + 4.8x - 2.5$



34. $g(x) = -\frac{2}{3}x^2 + \frac{5}{12}x + \frac{11}{6}$



Determine values for m and n such that the quadratic function has the given vertex.

35. $f(x) = mx^2 + 6x + n$ $V(-3, -4)$

36. $g(x) = mx^2 - 5nx + n$ $V(5, -23)$

37. $f(x) = -2x^2 + 6nx + m - 7n$ $V(3, -7)$

38. $g(x) = x^2 + 2mx + n$ $V(2, 3)$

39. $f(x) = mx^2 + (n-3)x + 2n$ $V(5, 15)$

40. $g(x) = 2mx^2 + nx$ $V(3, 6)$

41. $f(x) = x^2 + mx + 4n$ $V(-4, 0)$

42. $g(x) = -\frac{1}{4}mx^2 + 2nx + n$ $V(-2, 2)$

Write a quadratic function whose graph passes through the given set of points.

43. $(0, 6), (6, 12)$, and $(-2, 20)$

44. $(2, -1), (-1, 11)$, and $(0, 1)$

45. $(2, 3), (5, -3)$, and $(0, -13)$

46. $(6, -2), (12, 10)$, and $(3, 1)$

47. $(-2, -12), (0, 0)$, and $(-7, -7)$

48. $(-2, 5), (2, 3)$, and $(-4, 15)$

49. $(-1, -5), (-2, 10)$, and $\left(\frac{1}{2}, -\frac{35}{4}\right)$

50. $\left(-4, -\frac{7}{2}\right), \left(6, -\frac{17}{2}\right)$, and $\left(1, \frac{1}{4}\right)$

Determine the x- and y-intercepts of each quadratic function.

51. $f(x) = (x-4)(x+9)$

52. $g(x) = -3(x+2)(5x-3)$

53. $f(x) = x^2 - 9x + 20$

54. $g(x) = x^2 + x - 12$

55. $f(x) = x^2 + 12x + 36$

56. $g(x) = x^2 - 8x - 33$

57. $f(x) = 2x^2 + x - 15$

58. $g(x) = 9x^2 - 1$

59. $f(x) = 28x^2 - 33x - 28$

60. $g(x) = 16x^2 - 8x + 1$

Write a quadratic function in standard form given its roots.

61. $x = 5, -2$

62. $x = -1, -6$

63. $x = \frac{2}{3}, \frac{7}{4}$

64. $x = 0, -\frac{3}{2}$

65. $x = 0, -10$

66. $x = -\frac{3}{5}, \frac{1}{3}$

67. $x = \pm 2$

68. $x = \pm \frac{3}{5}$

In Exercises 67-71, describe and correct the error in each problem.

69. For the function, $f(x) = 3x^2 - 4x - 2$, the x-coordinate of the vertex is

$$x = \frac{b}{2a} = \frac{-4}{2(3)} = -\frac{4}{6} = -\frac{2}{3}.$$

70. For the function, $f(x) = 3x^2 - 4x - 2$, the y-intercept of the graph is the value of c, which is 2.

71. For the function, $f(x) = 3x^2 - 4x - 2$, if the x-coordinate of the vertex is $x = \frac{2}{3}$, then the y-coordinate of the vertex is $f\left(\frac{2}{3}\right)$.

$$y = f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) - 2$$
$$= 3\left(\frac{4}{3}\right) - \frac{8}{3} - 2$$
$$= 4 - \frac{8}{3} - 2$$
$$= -\frac{2}{3}$$

72. For the function, $g(x) = -x^2 - 4x + 3$, the x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{-4}{2(-1)} = 2.$$

73. For the function, $g(x) = -x^2 - 4x + 3$, if the x-coordinate of the vertex is $x = -2$, then the y-coordinate of the vertex is

$$y = g(-2) = -(-2)^2 - 4(2) + 3$$
$$= 4 - 8 + 3$$
$$= -1$$

74. A quadratic function is increasing when $x < -3$ and decreasing when $x > -3$. Is the vertex the highest or lowest point on the parabola? Explain your answer.

75. A quadratic function is decreasing when $x < 5$ and increasing when $x > 5$. Is the vertex the highest or lowest point on the parabola? Explain your answer.

76. The graph of which function has the same axis of symmetry as the graph of $y = x^2 - 10x + 3$?

a. $y = -x^2 - 10x + 5$

b. $y = 3x^2 + 30x - 22$

c. $y = -3x^2 + 30x + 22$

d. $y = 0.5x^2 - 4x + 3$

77. The graph of which function has the same axis of symmetry as the graph of $y = -2x^2 + 12x + 7$?

a. $y = x^2 - 8x + 6$

b. $y = -x^2 - 8x + 6$

c. $y = -x^2 - 6x + 8$

d. $y = x^2 - 6x + 8$

78. Which function represents the widest parabola? Explain your answer.

a. $y = 3x^2 - 10$

b. $y = -0.25x^2 + 0.7x - 1$

c. $y = -x^2 + 15x$

d. $y = 2x^2 - 10x + 3$

79. Given the x- and y-intercepts of the graph of a quadratic function, is it possible to determine the equation for the axis of symmetry? Explain your answer.

80. Write two different quadratic functions whose graphs have the axis of symmetry, $x = -4$.

81. The point $P(2, -6)$ lies on the graph of a quadratic function. Can $V(0, 2)$ be the vertex of the graph of the function? Explain your answer.

82. Determine the x- and y-intercepts in terms of m, n, and p for the quadratic function $f(x) = m(x - n)(x - p)$.

83. Determine the axis of symmetry in terms of m, n, and p for the quadratic function

$$f(x) = m(x - n)(x - p).$$

84. For the quadratic function, $f(x) = ax^2 + bx + c$, if $a < 0$, then determine the intervals on which the graph of the function is increasing and decreasing.

85. Write the quadratic function whose$x$-intercepts are $(6, 0)$ and $(-2, 0)$ and passes through $(-4, -5)$.