Quadratic Functions in Standard Form (ALG.QUAD.01)

1. Use the quadratic function $f(x) = 5x^2 - 7x - 2$ to answer each part.

a. Determine the leading coefficient of the function. **5**

b. What is the linear term? $\mathbf{-7x}$

c. What does the constant indicate? **the y-intercept, $(0, -2)$**

d. Does the graph of f open upward or downward? Explain your answer.
the graph opens upward because $a > 0$

e. How many x-intercepts will the graph of f have? Explain your answer.
the vertex is $(0.7, -4.45)$ and the graph opens upward, so there are two x-intercepts

2. Use the quadratic function $g(x) = -\frac{1}{3}x^2 + x - 6$ to answer each part.

a. Determine the leading coefficient of the function. $\mathbf{-\frac{1}{3}}$

b. What does the leading coefficient of the function indicate for the graph of g?
a stretch along the x-axis and also a reflection over the x-axis

c. What is the quadratic term? $\mathbf{-\frac{1}{3}x^2}$

d. Determine the y-intercept of the graph. $\mathbf{(0, -6)}$

e. How many x-intercepts will the graph of g have? Explain your answer.
the vertex is $\left(\frac{3}{2}, -\frac{21}{4}\right)$ and the graph opens downward, so there are no x-intercepts

For each quadratic function, determine (i) the vertex, (ii) whether the vertex is a maximum or minimum value of the function, (iii) whether the parabola opens upward or downward, (iv) the domain and range, (v) the axis of symmetry, and (vi) on what intervals the graph of the function is increasing and decreasing.

3. $f(x) = x^2 - 4x$

(i) $V(2, -4)$

(ii) the vertex is a minimum

(iii) the parabola opens upward

(iv) D: $(-\infty, +\infty)$ **R:** $[-4, +\infty)$

(v) $x = 2$

(vi) increasing: $(2, +\infty)$
decreasing: $(-\infty, 2)$

4. $g(x) = -3x^2 + 5$

(i) $V(0, 5)$

(ii) the vertex is a maximum

(iii) the parabola opens downward

(iv) D: $(-\infty, +\infty)$ **R:** $(-\infty, 5]$

(v) $x = 0$

(vi) increasing: $(-\infty, 0)$
decreasing: $(0, +\infty)$

5. $f(x) = 9 - x^2$

(i) $V(0, 9)$

(ii) the vertex is a maximum

(iii) the parabola opens downward

(iv) D: $(-\infty, +\infty)$ R: $(-\infty, 9]$

(v) $x = 0$

(vi) increasing: $(-\infty, 0)$
decreasing: $(0, +\infty)$

6. $g(x) = 21 - 20x + 10x^2$

(i) $V(1, 11)$

(ii) the vertex is a minimum

(iii) the parabola opens upward

(iv) D: $(-\infty, +\infty)$ R: $[11, +\infty)$

(v) $x = 1$

(vi) increasing: $(1, +\infty)$
decreasing: $(-\infty, 1)$

7. $f(x) = \frac{1}{2}x^2 - 5x + 9$

(i) $V(5, -3.5)$

(ii) the vertex is a minimum

(iii) the parabola opens upward

(iv) D: $(-\infty, +\infty)$ R: $[-3.5, +\infty)$

(v) $x = 5$

(vi) increasing: $(5, +\infty)$
decreasing: $(-\infty, 5)$

8. $g(x) = -0.75x^2 - 1.8x + 4.5$

(i) $V(-1.2, 5.58)$

(ii) the vertex is a maximum

(iii) the parabola opens downward

(iv) D: $(-\infty, +\infty)$ R: $(-\infty, 5.58]$

(v) $x = -1.2$

(vi) increasing: $(-\infty, -1.2)$
decreasing: $(-1.2, +\infty)$

9. $f(x) = -\frac{2}{3}x^2 + \frac{6}{5}x - \frac{8}{15}$

(i) $V\left(\frac{9}{10}, \frac{1}{150}\right)$

(ii) the vertex is a maximum

(iii) the parabola opens downward

(iv) D: $(-\infty, +\infty)$ R: $\left(-\infty, \frac{1}{150}\right]$

(v) $x = \frac{9}{10}$

(vi) increasing: $\left(-\infty, \frac{9}{10}\right)$
decreasing: $\left(\frac{9}{10}, +\infty\right)$

10. $g(x) = 0.1x^2 + 1.2x - 0.6$

(i) $V(-6, -4.2)$

(ii) the vertex is a minimum

(iii) the parabola opens upward

(iv) D: $(-\infty, +\infty)$ R: $[-4.2, +\infty)$

(v) $x = -6$

(vi) increasing: $(-6, +\infty)$
decreasing: $(-\infty, -6)$

Evaluate each quadratic function for the given values of x.

11. $f(x) = x^2 - 5x + 2$

$x = 0$	$x = -2$	$x = -\frac{1}{2}$	$x = \sqrt{5}$
$\mathbf{f(0) = 2}$	$\mathbf{f(-2) = 16}$	$\mathbf{f\left(-\frac{1}{2}\right) = \frac{19}{4}}$	$\mathbf{f(\sqrt{5}) = 7 - 5\sqrt{5}}$

12. $g(x) = 16 - x^2$

$x = 0$	$x = -4$	$x = 2.5$	$x = -2\sqrt{3}$
$\mathbf{g(0) = 16}$	$\mathbf{g(-4) = 0}$	$\mathbf{g(2.5) = 9.75}$	$\mathbf{g(-2\sqrt{3}) = 4}$

13. $f(x) = 3x^2 - 7x + 2$

$x = 1$	$x = \frac{1}{3}$	$x = 0.75$	$x = \sqrt{6}$
$\mathbf{f(1) = -2}$	$\mathbf{f\left(\frac{1}{3}\right) = 0}$	$\mathbf{f(0.75) = -1.5625}$	$\mathbf{f(\sqrt{6}) = 20 - 7\sqrt{6}}$

14. $g(x) = -\frac{3}{4}x^2 - \frac{3}{2}x + \frac{1}{3}$

$x = -2$	$x = \frac{8}{3}$	$x = -0.5$	$x = \frac{\sqrt{13}}{3} - 1$
$\mathbf{g(-2) = \frac{1}{3}}$	$\mathbf{g\left(\frac{8}{3}\right) = -9}$	$\mathbf{g(-0.5) = \frac{43}{48}}$	$\mathbf{g\left(\frac{\sqrt{13}}{3} - 1\right) = 0}$

In Exercises 15–20, match the quadratic function with its graph. The graphs are labeled (a), (b), (c), (d), (e), and (f).

15. $f(x) = x^2 - 4x + 3$ **e**

16. $f(x) = x^2 - 4x + 5$ **c**

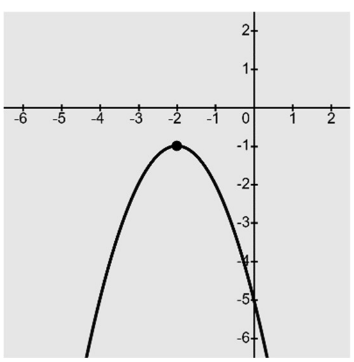
17. $f(x) = x^2 + 4x + 5$ **b**

18. $f(x) = -x^2 - 4x - 3$ **d**

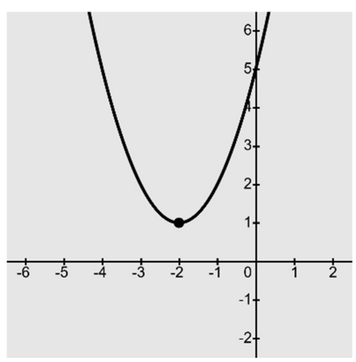
19. $f(x) = -x^2 + 4x - 5$ **f**

20. $f(x) = -x^2 - 4x - 5$ **a**

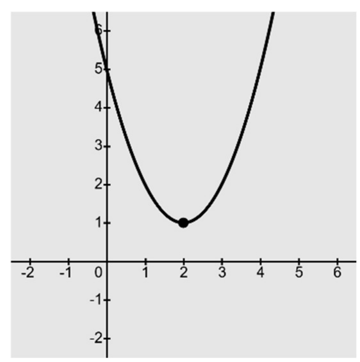
(a)



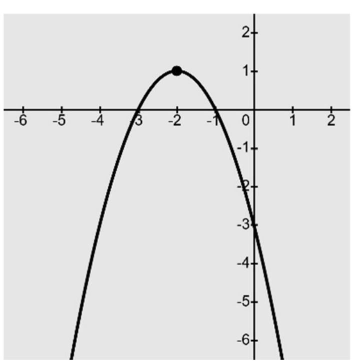
(b)



(c)



(d)



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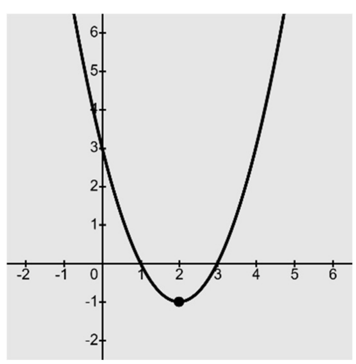
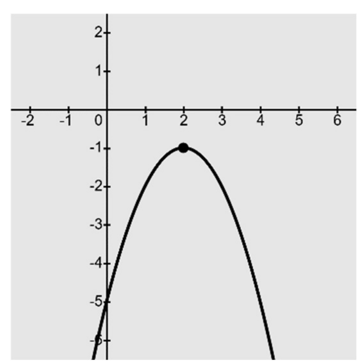


(f)



Convert each quadratic function to standard form.

21. $f(x) = (x-2)^2 + 3$

$\boldsymbol{f(x) = x^2 - 4x + 7}$

22. $g(x) = (x+6)^2 - 24$

$\boldsymbol{g(x) = x^2 + 12x + 12}$

23. $f(x) = (6x+1)^2 - 1$

$\boldsymbol{f(x) = 36x^2 + 12x}$

24. $g(x) = (2x-5)^2 - 8$

$\boldsymbol{g(x) = 4x^2 - 20x + 17}$

25. $f(x) = -7\left(\frac{1}{2}x - 3\right)^2$

$\boldsymbol{f(x) = -\frac{7}{4}x^2 + 21x - 63}$

26. $g(x) = 4(x+2)^2$

$\boldsymbol{g(x) = 4x^2 + 16x + 16}$

27. $f(x) = \frac{2}{3}(3x-1)^2 + 6$

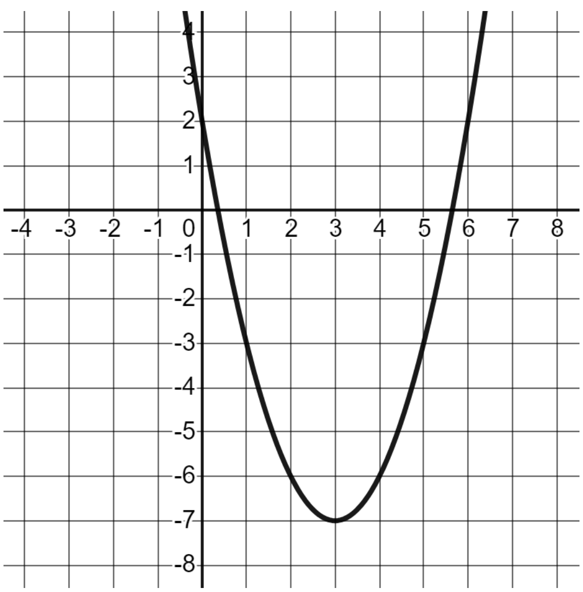
$\boldsymbol{f(x) = 6x^2 - 4x + \frac{20}{3}}$

28. $g(x) = -\frac{1}{5}(x+5)^2 + 7$

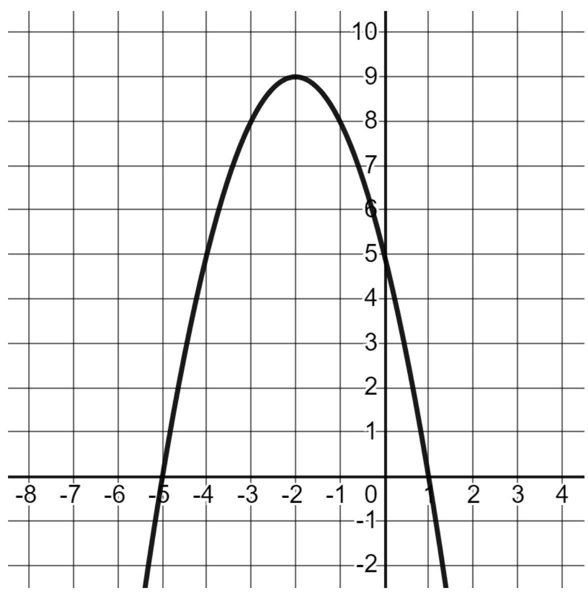
$\boldsymbol{g(x) = -\frac{1}{5}x^2 - 2x + 2}$

Graph each quadratic function by first finding its vertex and completing a table of values.

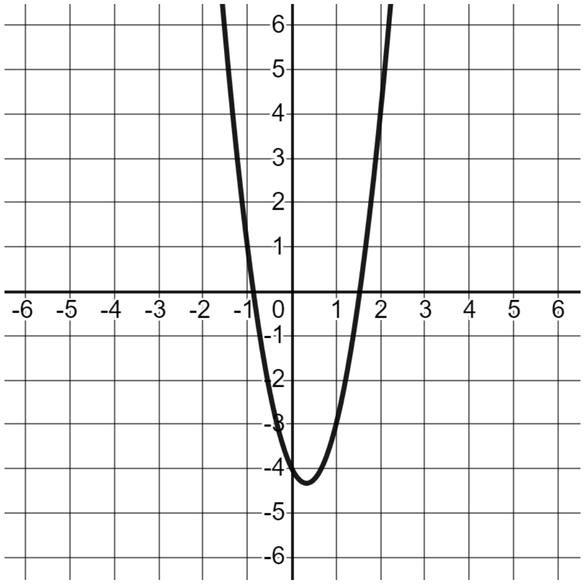
29. $f(x) = x^2 - 6x + 2$



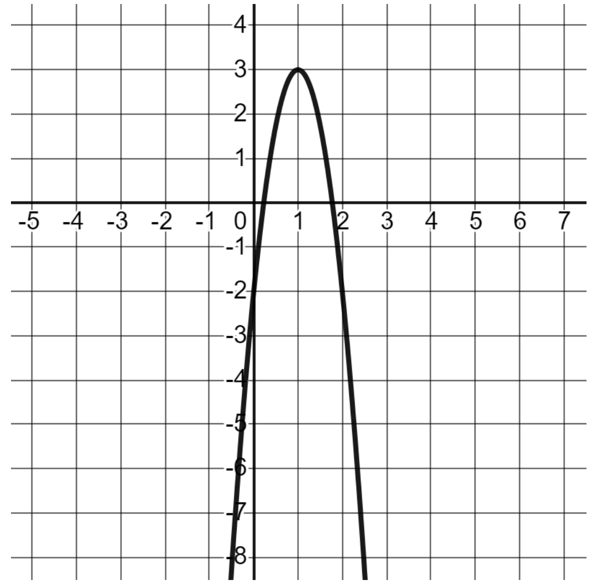
30. $g(x) = -x^2 - 4x + 5$



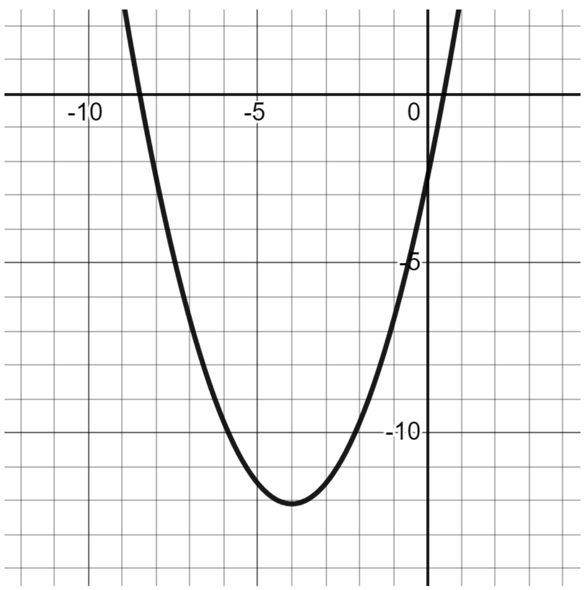
31. $f(x) = 3x^2 - 2x - 4$



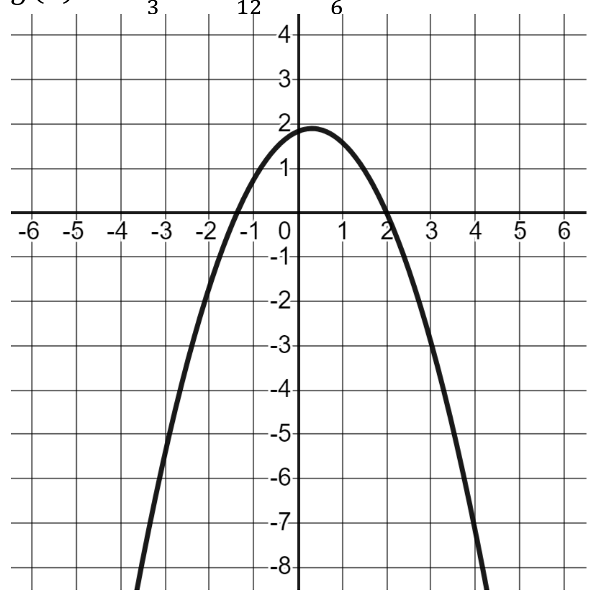
32. $g(x) = -5x^2 + 10x - 2$



33. $f(x) = 0.6x^2 + 4.8x - 2.5$



34. $g(x) = -\frac{2}{3}x^2 + \frac{5}{12}x + \frac{11}{6}$



Determine values for m and n such that the quadratic function has the given vertex.

35. $f(x) = mx^2 + 6x + n$ $V(-3,-4)$

$\boldsymbol{m = 1,\ n = 5}$

36. $g(x) = mx^2 - 5nx + n$ $V(5,-23)$

$\boldsymbol{m = 1,\ n = 2}$

37. $f(x) = -2x^2 + 6nx + m - 7n$ $V(3,-7)$

$\boldsymbol{m = -11,\ n = 2}$

38. $g(x) = x^2 + 2mx + n$ $V(2,3)$

$\boldsymbol{m = -2,\ n = 7}$

39. $f(x) = mx^2 + (n-3)x + 2n$ $V(5,15)$

$\boldsymbol{m = -\frac{1}{5},\ n = 5}$

40. $g(x) = 2mx^2 + nx$ $V(3,6)$

$\boldsymbol{m = -\frac{1}{3},\ n = 4}$

41. $f(x) = x^2 + mx + 4n$ $V(-4,0)$

$\boldsymbol{m = 8,\ n = 4}$

42. $g(x) = -\frac{1}{4}mx^2 + 2nx + n$ $V(-2,2)$

$\boldsymbol{m = 4,\ n = -2}$

Write a quadratic function whose graph passes through the given set of points.

43. $(0,6), (6,12)$, and $(-2,20)$

$\boldsymbol{f(x) = x^2 - 5x + 6}$

44. $(2,-1), (-1,11)$, and $(0,1)$

$\boldsymbol{f(x) = 3x^2 - 7x + 1}$

45. $(2,3), (5,-3)$, and $(0,-13)$

$\boldsymbol{f(x) = -2x^2 + 12x - 13}$

46. $(6,-2), (12,10)$, and $(3,1)$

$\boldsymbol{f(x) = \frac{1}{3}x^2 - 4x + 10}$

47. $(-2,-12), (0,0)$, and $(-7,-7)$

$\boldsymbol{f(x) = x^2 + 8x}$

48. $(-2,5), (2,3)$, and $(-4,15)$

$\boldsymbol{f(x) = \frac{3}{4}x^2 - \frac{1}{2}x + 1}$

49. $(-1,-5), (-2,10)$, and $\left(\frac{1}{2}, -\frac{35}{4}\right)$

$\boldsymbol{f(x) = 5x^2 - 10}$

50. $\left(-4, -\frac{7}{2}\right), \left(6, -\frac{17}{2}\right)$, and $\left(1, \frac{1}{4}\right)$

$\boldsymbol{f(x) = -\frac{1}{4}x^2 + \frac{1}{2}}$

Determine the x- and y-intercepts of each quadratic function.

51. $f(x) = (x-4)(x+9)$
$\mathbf{(4,0), (-9,0), (0,-36)}$

52. $g(x) = -3(x+2)(5x-3)$
$\mathbf{(-2,0), \left(\frac{3}{5}, 0\right), (0,18)}$

53. $f(x) = x^2 - 9x + 20$
$\mathbf{(5,0), (4,0), (0,20)}$

54. $g(x) = x^2 + x - 12$
$\mathbf{(3,0), (-4,0), (0,-12)}$

55. $f(x) = x^2 + 12x + 36$
$\mathbf{(-6,0), (0,36)}$

56. $g(x) = x^2 - 8x - 33$
$\mathbf{(11,0), (-3,0), (0,-33)}$

57. $f(x) = 2x^2 + x - 15$
$\mathbf{(-3,0), \left(\frac{5}{2}, 0\right), (0,-15)}$

58. $g(x) = 9x^2 - 1$
$\mathbf{\left(\frac{1}{3}, 0\right), \left(-\frac{1}{3}, 0\right), (0,-1)}$

59. $f(x) = 28x^2 - 33x - 28$
$\mathbf{\left(\frac{7}{4}, 0\right), \left(-\frac{4}{7}, 0\right), (0,-28)}$

60. $g(x) = 16x^2 - 8x + 1$
$\mathbf{\left(\frac{1}{4}, 0\right), (0,1)}$

Write a quadratic function in standard form given its roots.

61. $x = 5, -2$ $\quad \mathbf{f(x) = x^2 - 3x - 10}$

62. $x = -1, -6$ $\quad \mathbf{f(x) = x^2 + 7x + 6}$

63. $x = \frac{2}{3}, \frac{7}{4}$ $\quad \mathbf{f(x) = 12x^2 - 29x + 14}$

64. $x = 0, -\frac{3}{2}$ $\quad \mathbf{f(x) = 2x^2 + 3x}$

65. $x = 0, -10$ $\quad \mathbf{f(x) = x^2 + 10x}$

66. $x = -\frac{3}{5}, \frac{1}{3}$ $\quad \mathbf{f(x) = 15x^2 + 4x - 3}$

67. $x = \pm 2$ $\quad \mathbf{f(x) = x^2 - 4}$

68. $x = \pm\frac{3}{5}$ $\quad \mathbf{f(x) = 25x^2 - 9}$

In Exercises 67-71, describe and correct the error in each problem.

69. For the function, $f(x) = 3x^2 - 4x - 2$, the x-coordinate of the vertex is

$$x = \frac{b}{2a} = \frac{-4}{2(3)} = -\frac{4}{6} = -\frac{2}{3}.$$

The x-value of the vertex is $x = -\frac{b}{2a}$.
The error is the missing negative sign.

70. For the function, $f(x) = 3x^2 - 4x - 2$, the y-intercept of the graph is the value of c, which is 2.

The y-intercept is c, but the value of c in the function is -2.

71. For the function, $f(x) = 3x^2 - 4x - 2$, if the x-coordinate of the vertex is $x = \frac{2}{3}$, then the y-coordinate of the vertex is $f\left(\frac{2}{3}\right)$.

$$y = f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) - 2$$
$$= 3\left(\frac{4}{3}\right) - \frac{8}{3} - 2$$
$$= 4 - \frac{8}{3} - 2$$
$$= -\frac{2}{3}$$

When squaring the $\frac{2}{3}$, only the numerator was squared. It should be $\frac{4}{9}$.

72. For the function, $g(x) = -x^2 - 4x + 3$, the x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{-4}{2(-1)} = 2.$$

There are three negative signs in the final step, so the answer should be -2, not 2.

73. For the function, $g(x) = -x^2 - 4x + 3$, if the x-coordinate of the vertex is $x = -2$, then the y-coordinate of the vertex is

$$y = g(-2) = -(-2)^2 - 4(2) + 3$$
$$= 4 - 8 + 3$$
$$= -1$$

When squaring $-(-2)^2$, it should be -4, not 4.

74. A quadratic function is increasing when $x < -3$ and decreasing when $x > -3$. Is the vertex the highest or lowest point on the parabola? Explain your answer.

If the function is increasing when $x < -3$ and decreasing when $x > -3$, then the parabola opens downward and the vertex is a maximum of the function.

75. A quadratic function is decreasing when $x < 5$ and increasing when $x > 5$. Is the vertex the highest or lowest point on the parabola? Explain your answer.

If the function is decreasing when $x < 5$ and increasing when $x > 5$, then the parabola opens upward and the vertex is a minimum of the function.

76. The graph of which function has the same axis of symmetry as the graph of $y = x^2 - 10x + 3$? **C**

a. $y = -x^2 - 10x + 5$

b. $y = 3x^2 + 30x - 22$

c. $y = -3x^2 + 30x + 22$

d. $y = 0.5x^2 - 4x + 3$

77. The graph of which function has the same axis of symmetry as the graph of $y = -2x^2 + 12x + 7$? **D**

a. $y = x^2 - 8x + 6$

b. $y = -x^2 - 8x + 6$

c. $y = -x^2 - 6x + 8$

d. $y = x^2 - 6x + 8$

78. Which function represents the widest parabola? Explain your answer.

a. $y = 3x^2 - 10$

b. $y = -0.25x^2 + 0.7x - 1$

c. $y = -x^2 + 15x$

d. $y = 2x^2 - 10x + 3$

B. The absolute value of a, the leading coefficient, is the closest to zero.

79. Given the x- and y-intercepts of the graph of a quadratic function, is it possible to determine the equation for the axis of symmetry? Explain your answer.

If the function has 0 x-intercepts, then it is not possible to determine the axis of symmetry using intercepts alone. If the function has 1 x-intercept, then the axis of symmetry passes through that x-intercept. If the function has 2 x-intercepts, then the axis of symmetry will pass through the midpoint of the two intercepts.

80. Write two different quadratic functions whose graphs have the axis of symmetry, $x = -4$.

Answers will vary. Example,
$f(x) = x^2 + 8x + 16$ **or**
$f(x) = -2x^2 - 16x + 3$

81. The point $P(2, -6)$ lies on the graph of a quadratic function. Can $V(0, 2)$ be the vertex of the graph of the function? Explain your answer.

Yes. Any ordered pair whose x-value is not 2 could be the vertex.

82. Determine the x- and y-intercepts in terms of m, n, and p for the quadratic function

$$f(x) = m(x - n)(x - p).$$

$(n, 0), (p, 0), (0, mnp)$

83. Determine the axis of symmetry in terms of m, n, and p for the quadratic function

$$f(x) = m(x - n)(x - p).$$

$$x = \frac{n + p}{2}$$

84. For the quadratic function, $f(x) = ax^2 + bx + c$, if $a < 0$, then determine the intervals on which the graph of the function is increasing and decreasing.

Increasing where $x < -\frac{b}{2a}$ and decreasing where $x > -\frac{b}{2a}$.

85. Write the quadratic function whose x-intercepts are $(6, 0)$ and $(-2, 0)$ and passes through $(-4, -5)$.

$$f(x) = -\frac{1}{4}x^2 + x + 3$$