

Quadratic Functions in Vertex Form (ALG.QUAD.02)

For each quadratic function, determine (i) the vertex, (ii) whether the vertex is a maximum or minimum value of the function, (iii) whether the parabola opens upward or downward, (iv) the domain and range, (v) the axis of symmetry, and (vi) on what intervals the graph of the function is increasing and decreasing.

1. $f(x) = (x - 2)^2 + 6$

2. $g(x) = (x + 5)^2 - 3$

3. $f(x) = -3x^2 + 4$

4. $g(x) = \frac{1}{2}x^2 - 5$

5. $f(x) = -(x - 4)^2 - 2$

6. $g(x) = 2(x + 1)^2$

7. $f(x) = \frac{3}{4}(x + 2)^2$

8. $g(x) = -\frac{5}{3}(x - 6)^2 + 3$

Evaluate each quadratic function for the given values of x .

9. $f(x) = -3(x - 4)^2 + 1$ $x = 0$ $x = 2$ $x = -1$ $x = 6$

10. $g(x) = 5(x + 2)^2 - 3$ $x = 0$ $x = -\frac{3}{5}$ $x = 0.7$ $x = -2$

11. $f(x) = \frac{1}{4}(x + 8)^2 + 5$ $x = -10$ $x = 2$ $x = -\frac{17}{3}$ $x = \sqrt{3} - 6$

12. $g(x) = -\frac{7}{2}(x - 6)^2 - 2$ $x = 2$ $x = -\frac{3}{4}$ $x = -0.4$ $x = 3 + \sqrt{2}$

Determine the x - and y -intercepts of each quadratic function.

13. $f(x) = (x + 3)^2$

14. $g(x) = (x - 1)^2 - 4$

15. $f(x) = (x - 7)^2 + 1$

16. $g(x) = 2(6x - 1)^2 - 10$

17. $f(x) = 3(x + 5)^2 - 27$

18. $g(x) = -(2x - 1)^2 - 1$

19. $f(x) = -\frac{5}{4}(x + 3)^2 + 5$

20. $g(x) = \frac{1}{2}(5x - 8)^2 - 3$

In Exercises #21–26, match the quadratic function with its graph. The graphs are labeled (a), (b), (c), (d), (e), and (f).

21. $f(x) = 2(x - 1)^2 + 1$

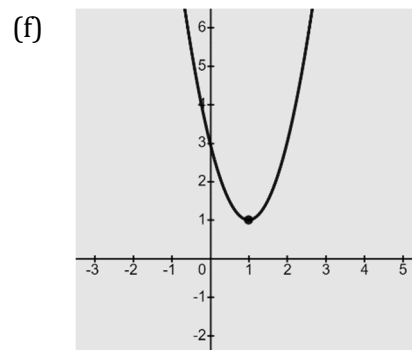
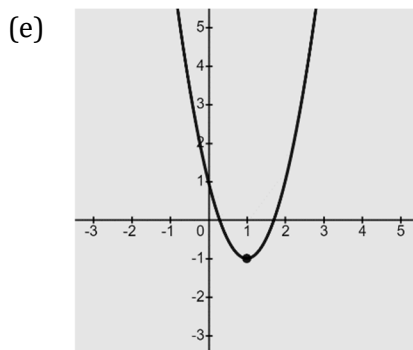
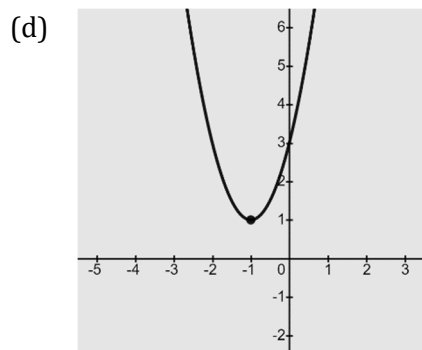
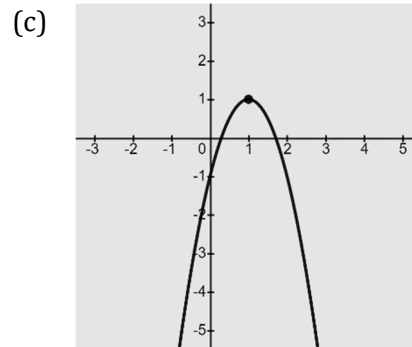
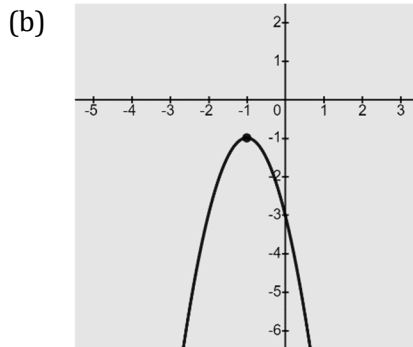
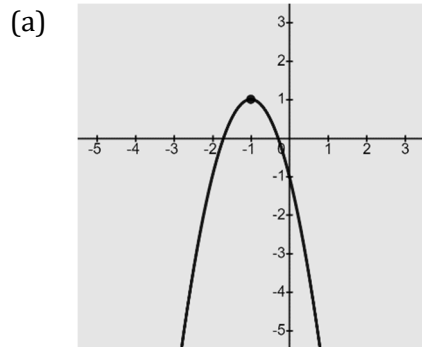
22. $f(x) = 2(x + 1)^2 + 1$

23. $f(x) = -2(x - 1)^2 + 1$

24. $f(x) = -2(x + 1)^2 + 1$

25. $f(x) = 2(x - 1)^2 - 1$

26. $f(x) = -2(x + 1)^2 - 1$



Describe the transformation of each graph from its quadratic parent function. Then identify the vertex.

27. $f(x) = -(x - 1)^2 + 3$

28. $g(x) = \frac{1}{2}(x + 4)^2$

29. $f(x) = -3x^2 + 2$

30. $g(x) = (x - 5)^2 - 2$

31. $f(x) = -2.5(x - 0.75)^2 + 1.25$

32. $g(x) = \frac{7}{3}(x + 3)^2 - 11$

Write a quadratic function for each transformation from the parent function $f(x) = x^2$.

33. a horizontal shift four units right and a vertical shift two units down

34. a reflection over the x -axis and a compression by a factor of three

35. a reflection over the x -axis and a vertical shift five units up

36. a stretch by a factor of two and a horizontal shift six units left

In Exercises #37–46, write a quadratic function in vertex form for each parabola.

37. the parabola passes through $(2, 4)$ and has a vertex at $(0, -4)$

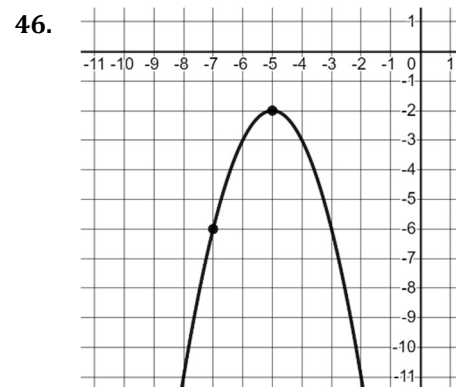
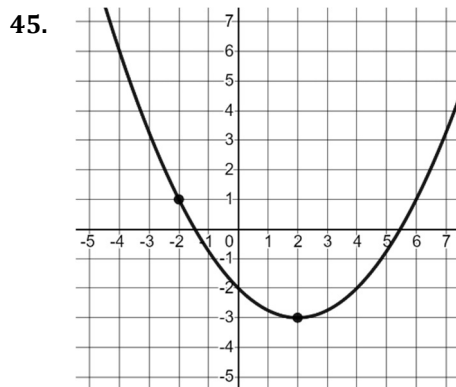
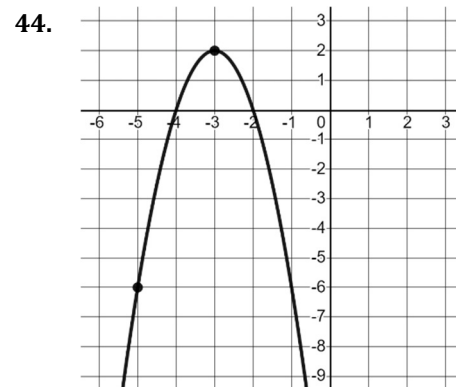
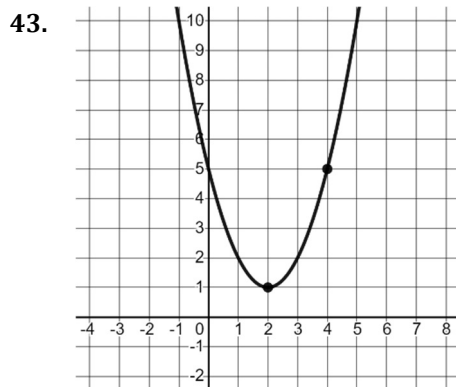
38. the parabola passes through $(0, 10)$ and has a vertex at $(3, 1)$

39. the parabola passes through $(-6, 0)$ and has a vertex at $(-3, 3)$

40. the parabola passes through $(-7, -11)$ and has a vertex at $(-4, -2)$

41. the parabola passes through $(12, 7)$ and has a vertex at $(4, -9)$

42. the parabola passes through $(15, -22)$ and has a vertex at $(5, 8)$



Solve each quadratic equation by using the square root property.

47. $(x - 3)^2 = 25$

48. $(5x + 2)^2 = 49$

49. $3(x + 6)^2 - 108 = 0$

50. $-4(x - 5)^2 + 196 = 0$

51. $\frac{1}{3}(3x - 1)^2 - 12 = 0$

52. $3.5(x + 2.5)^2 - 70.875 = 0$

For each quadratic function, (i) determine the domain and range, (ii) determine the inverse function, $f^{-1}(x)$, (iii) show algebraically that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$, and (iv) determine the domain and range of $f^{-1}(x)$.

53. $f(x) = (x + 6)^2$

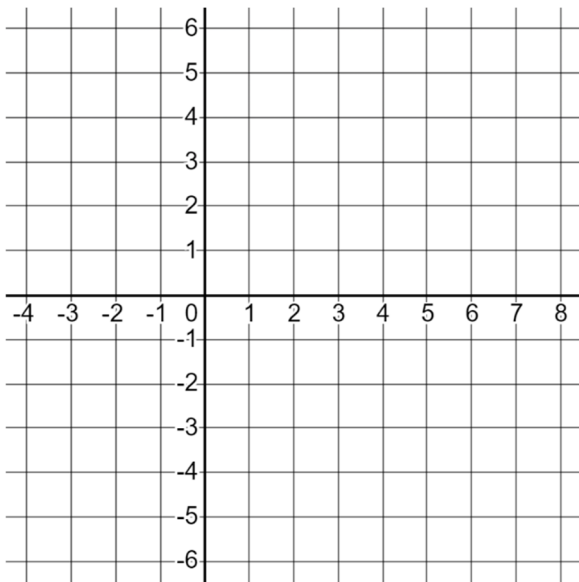
54. $f(x) = -(x - 5)^2 + 2$

55. $f(x) = -2(x + 4)^2$

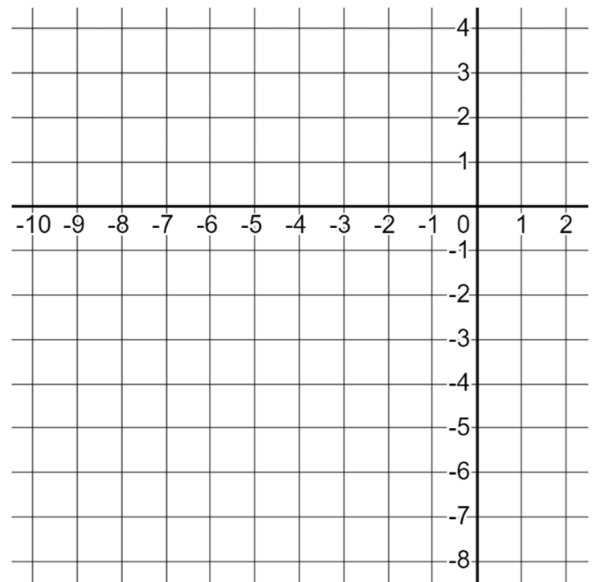
56. $f(x) = \frac{1}{3}(5x - 2)^2 + 1$

Graph each quadratic function by first finding its vertex and then using a table of values.

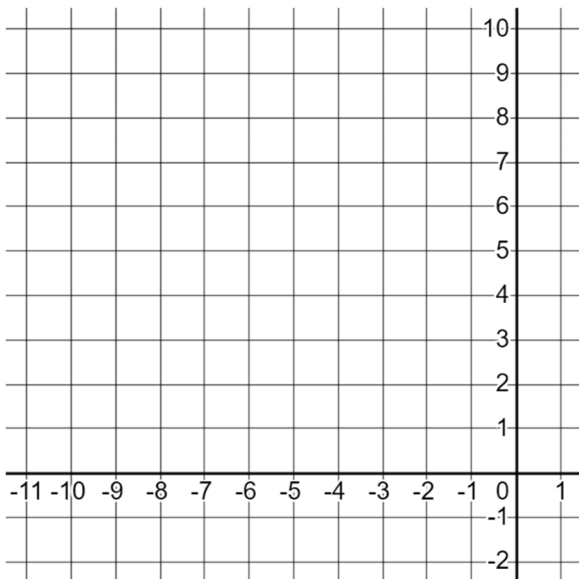
57. $f(x) = (x - 2)^2 - 5$



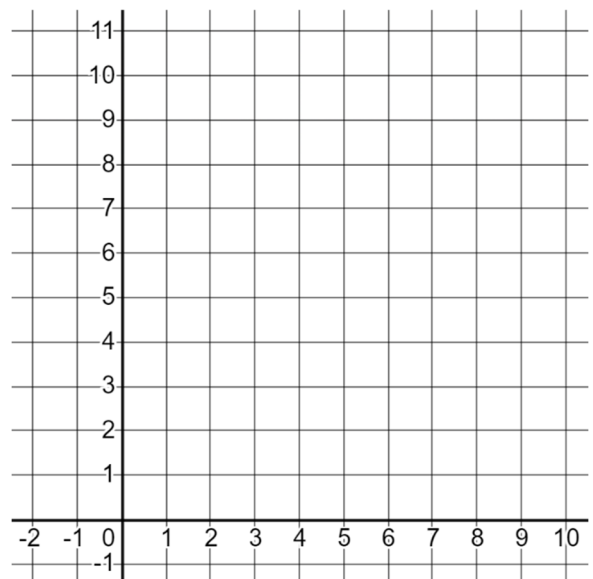
58. $g(x) = -(x + 4)^2 + 3$



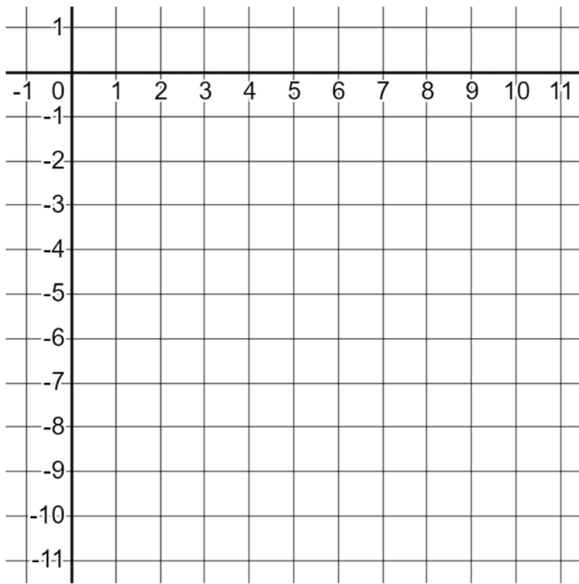
59. $f(x) = 3(x + 5)^2 - 1$



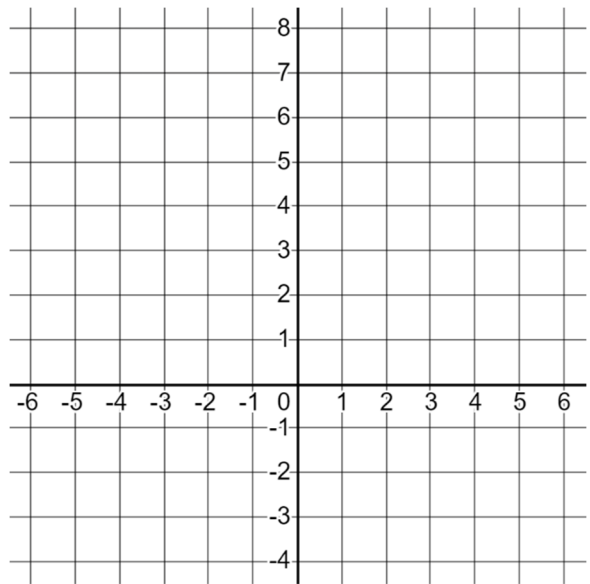
60. $g(x) = \frac{1}{2}(x - 4)^2 + 3$



61. $f(x) = -\frac{2}{3}(x - 6)^2$



62. $g(x) = -\frac{5}{4}x^2 + 6$



63. Compare the following three functions both graphically and algebraically. What do you notice? Explain your answer.

$$f(x) = (x + 2)(x + 6)$$

$$g(x) = (x + 4)^2 - 4$$

$$h(x) = x^2 + 8x + 12$$

66. A quadratic function has x -intercepts at $(-8, 0)$ and $(4, 0)$. Its range is $[-4, +\infty)$. What is the vertex of the function? Write the function.

68. Determine a quadratic function that is increasing on the interval $(-\infty, -2)$, decreasing on the interval $(-2, +\infty)$, is positive on the interval $(-3, -1)$, and has a y -intercept at $(0, -12)$.

70. For the quadratic function, $f(x) = (x - h)^2$, how many x - and y -intercepts will the graph have? Explain your answer.

64. The point $P(3, 8)$ lies on the graph of a quadratic function. Can $Q(1, 8)$ be the vertex of the graph of the function? Explain your answer.

65. If $f(x) = a(x - b)^2 + c$, then determine $f^{-1}(x)$ in terms of a , b , and c .

67. A quadratic function has intercepts at $(3, 0)$ and $(0, -6)$. Determine both the function and its range.

69. Use the square root property to solve the quadratic equation for x , in terms of a , b , and c .

$$a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0$$