

Trigonometry

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

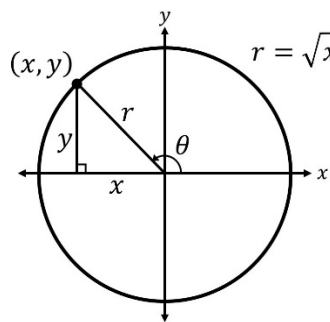
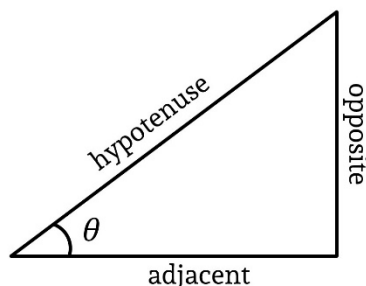
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$



$$r = \sqrt{x^2 + y^2}$$

Circular functions definitions, where θ is any angle.

$$\sin \theta = \frac{y}{r}$$

$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

	$\theta = 0^\circ$	$\theta = 15^\circ$	$\theta = 30^\circ$	$\theta = 45^\circ$	$\theta = 60^\circ$	$\theta = 75^\circ$	$\theta = 90^\circ$
sin θ	0	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	1
cos θ	1	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	0
tan θ	0	$2 - \sqrt{3}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2 + \sqrt{3}$	<i>undefined</i>

Reciprocal Functions

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Identities for Negatives

$$\sin(-x) = -\sin x$$

$$\csc(-x) = -\csc x$$

$$\cos(-x) = \cos x$$

$$\sec(-x) = \sec x$$

$$\tan(-x) = -\tan x$$

$$\cot(-x) = -\cot x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \cot x}{\cot^2 x - 1} = \frac{2}{\cot x - \tan x}$$

Triple-Angle Identities

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

Trigonometry

Half-Angle Identities

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$
$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Power-Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Sum and Difference Identities

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$
$$\sin x - \sin y = 2 \sin \left(\frac{x-y}{2} \right) \cos \left(\frac{x+y}{2} \right)$$
$$\cos x + \cos y = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$
$$\cos x - \cos y = -2 \sin \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)$$

Product-to-Sum Formulas

$$\sin x \cdot \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$
$$\cos x \cdot \cos y = \frac{1}{2} [\cos(x-y) + \cos(x+y)]$$
$$\sin x \cdot \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$
$$\cos x \cdot \sin y = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Heron's Formula (Area)

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}$$

Projectile Motion

position

$$s_y(t) = \frac{1}{2}gt^2 + (v_0 \cdot \sin \theta)t + s_0$$

$$s_x(t) = v_0 \cdot \cos \theta \cdot t$$

Gravity: $g = -9.8 \text{ m/sec}^2$ or $g = -32 \text{ ft/sec}^2$

velocity

$$v_y(t) = gt + v_0 \cdot \sin \theta$$

$$v_x(t) = v_0 \cdot \cos \theta$$

acceleration

$$a_y(t) = g$$

$$a_x(t) = 0$$

Inverse Trigonometric Functions

$$y = \sin^{-1} x \text{ means } x = \sin y$$

$$-1 \leq x \leq 1 \text{ and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$y = \cos^{-1} x \text{ means } x = \cos y$$

$$-1 \leq x \leq 1 \text{ and } 0 \leq y \leq \pi$$

$$y = \tan^{-1} x \text{ means } x = \tan y$$

$$x \in \mathbb{R} \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}$$